CPSC 365 / ECON 365: Algorithms Yale University

Discussion 1

Out: February 3, 2022 Discussed: February 4, 2022

1 Stable matching

Consider a set of n men $M = \{m_1, m_2, \ldots, m_n\}$, and a set of n women $W = \{w_1, w_2, \ldots, w_n\}$.

Suppose every $m_i \in M$ and every $w_i \in W$ have same the preference lists:

$$\text{preferences for each } m_i \in M\text{:} \quad w_1, w_2, w_3, \ldots, w_n$$
$$\text{preferences for each } w_i \in M\text{:} \quad m_1, m_2, m_3, \ldots, m_n \;.$$

Prove that there is a unique stable matching in this instance.

2 Graphs

2.1 Degree Sum Formula

Let $G = (V, E)$ be an undirected graph. The **degree** of a node $u \in V$, denoted by $d(u)$, is the number of neighbors that u has, or equivalently, the number of edges incident upon u. Show that

$$\sum_{u \in V} d(u) = 2|E|.$$

2.2 Handshaking Lemma

Suppose we have a party with n people. Any two people may shake hands, or not. We say a person is *odd* if they have shaken hands with an odd number of other people. Show that the number of odd people is even.