CPSC 365 / ECON 365: Algorithms	Yale University
Discussion 3	
Out: February 17, 2022	Discussed: February 18, 2022

1 Negative Edges

Let $G = (V, E)$ be a graph with edge weights $L = \{\ell_e : e \in E\}$. Each weight ℓ_e may be negative, but assume there are no negative cycles.

1. Assume that L contains at least one negative edge weight, and consider the following proposed modification to Dijkstra's algorithm:

   - Find the minimum edge weight in L, denoted by ℓ^*. By assumption, $\ell^* < 0$.
   - For all $e \in E$, let $\ell'_e := \ell_e + |\ell^*| + 1$ (meaning all $\ell'_e > 0$).
     Define a new set of edge weights $L' = \{\ell'_e : e \in E\}$.
     Run Dijkstra's algorithm as usual but using the edge weights L'.

   Prove or disprove: for any source vertex u and all other vertices $v \in V$, running the modified algorithm correctly computes the shortest path from u to v with respect to the original edge weights L.

2. Consider the same construction of the edge weights L' as in part (1).

   Prove or disprove: an MST with respect to the edge weights L' is also an MST with respect to the edge weights L.

2 MST True/False

The following statements may or may not be correct. For each statement, either prove it (if it is correct) or give a counterexample. Always assume that the graph $G = (V, E)$ is undirected. Do not assume that edge weights are distinct unless specifically stated.

1. If the graph G has more than $|V| - 1$ edges, and there is a unique heaviest edge, then this edge cannot be part of a minimum spanning tree.
2. Let $e = (u, v)$ be any edge of minimum weight in G. Then e must be part of some MST.
3. If the lightest edge in a graph is unique, then it must be part of every MST.
4. The shortest-path tree computed by Dijkstra's algorithm is necessarily an MST.