CPSC 365 / ECON 365: Algorithms Yale University

Discussion 5

Out: March 3, 2022 Discussed: March 4, 2022

1 Rod Cutting

You are given a metal rod of length n inches, for some integer $n \geq 1$. Treating the left endpoint of the rod as position 0, and the right endpoint as position n, you can cut the rod at any integer position $i = 1, \ldots, n-1$ at no cost. You can make cuts at multiple positions (or make no cuts at all). Every resulting piece will be sold at a predefined price depending on its length. These prices are given by a function P, where $P(i)$ is the price for which you can sell a rod of length i, for all $i = 1, \ldots, n$.

Given the function P, design a dynamic programming algorithm for determining the maximum achievable profit from cutting the rod and selling the resulting pieces.

Example:

Rod of length 4. $P = (1, 5, 8, 9)$.
Profit-maximizing strategy: cut at index 2, and sell two pieces of length 2.
Total profit is $2 \cdot P(2) = 2 \cdot 5 = 10$.

Problems:

(a) Define what the entries of your dynamic programming table are, in words. (E.g., $T(i)$ is $\ldots$). What is the final answer of your algorithm in terms of the table entries?

(b) State the recurrence for entries of the table in terms of smaller subproblems, and state the base case(s). (You don't have to give a formal proof, but explain why the recurrrence is correct).

(c) Write pseudocode for your algorithm to solve this problem.

(d) Analyze the running time of your algorithm.

2 Bottom-Up vs. Top-Down

Recall from class we can compute the Fibonacci numbers $F(n) = F(n-1)+F(n-2)$ in two different ways: Top-down (using recursion with memoization) and bottom-up (using dynamic programming).

In this case, we saw that they both have the same running time (although typically the constant involved in recursion is larger due to overhead), since they are computing the same subproblems in different ways.

Now suppose we have a sequence $G(n)$ satisfying:

$$G(n) = G(n - 1000) + G(n - 2000)$$

for all $n \geq 2$, with base cases $G(1) = 1$ and $G(n) = 0$ for $n \leq 0$. Suppose you want to compute $G(10^6)$. Which method would you use: bottom-up or top-down?

3 Running Bellman-Ford

Run the iterations of Bellman-Ford on the following graph, to compute the shortest distance from s to all vertices v.