CPSC 365 / ECON 365: Algorithms	Yale University
Discussion 6	
Out: March 31, 2022	Discussed: April 1, 2022

1 Flows and Cuts

(a) Given a flow network $G = (V, E)$ with all even capacities, prove that the size of the maximum flow is even.

(b) Let $G = (V, E)$ be a flow network with source and sink s, t, and suppose there exist edges $e = (u, v)$ and $e' = (v, u)$ in E. Show that there exists a maximum flow f where one of e or e' has no flow (either $f(e) = 0$ or $f(e') = 0$).

(c) You are given a flow network $G = (V, E)$ with source and sink s, t, and with positive integer capacities c_e for every $e \in E$. You are given an integer maximum $s - t$ flow f in G.

Suppose we pick a specific edge $e \in E$ and increase its capacity by one unit. Let G' be the resulting flow network. Show how to find a maximum flow in G' in $O(|V| + |E|)$ time.

2 Three Group Matching

Consider three groups of people M, W, L where $|M| = k$, $|W| = m$, and $|L| = n$. We want to construct a set of triples $Z \subseteq M \times W \times L$ of maximum size subject to the following constraints:

(1) Given sets $P_1 \subseteq M \times W$ and $P_2 \subseteq W \times L$, for all $(u, v, w) \in M \times W \times L$, if $(u, v, w) \in Z$ then $(u, v) \notin P_1$ and $(v, w) \notin P_2$.

In other words, P_1 prevents certain pairs of $M \times W$ from being included in Z, and P_2 prevents certain pairs of $W \times L$ from being included in Z.

(2) No element in M, W, L appears in more than 1 triplet in Z.

Design an algorithm for finding the size of the largest possible matching subject to these constraints by reducing the problem to Max Flow.