

## Discussion 7

Out: April 7, 2022

Discussed: April 8, 2022

## 1 STINGY-SAT is Hard

STINGY-SAT is the following decision problem: given a Boolean formula  $\phi$  in CNF and a positive integer  $k$ , return YES if there exists a satisfying assignment of  $\phi$  in which at most  $k$  variables are True, and NO otherwise. Prove that STINGY-SAT is **NP**-complete via the following two steps:

- (a) Show that STINGY-SAT is in **NP**
- (b) Prove that SAT reduces to STINGY-SAT, which implies that STINGY-SAT is **NP**-hard.

## 2 CLIQUE is Hard

CLIQUE is the following problem: given a graph  $G$  and an integer  $k \geq 2$ , return YES if  $G$  contains a clique of size  $k$ , and return NO otherwise. Recall that a clique is a subgraph  $S$  of  $G$  such that every pair of vertices in  $S$  shares an edge in  $G$ .

- (a) Show that CLIQUE is in **NP**
- (b) Prove that 3SAT reduces to CLIQUE, which implies that CLIQUE is **NP**-hard.