

Discussion 8

Out: April 14, 2022

Discussed: April 15, 2022

1 4-SAT

Consider the following problems:

- **4-SAT**: Given a Boolean formula in CNF form, where each clause has *exactly* 4 terms, determine whether there exists a satisfying assignment, and return it.
- **Almost-3-SAT**: Given a Boolean formula in CNF form, where each clause has *at most* 3 terms, determine whether there exists a satisfying assignment, and return it.

Note that the terms in each clause must be distinct. You may assume that **Almost-3-SAT** is NP-complete. Show that **4-SAT** is NP-complete, by reducing from **Almost-3-SAT**.

- (a) Give a polynomial time algorithm to verify solutions of **4-SAT**.
- (b) Reduce from **Almost-3-SAT**.
- (c) Show the correctness of your reduction.

2 2-SAT

Recall in class we saw that **2-SAT** can be solved via graph algorithms, by constructing a particular graph from a formula and labelling the vertices appropriately.

Consider the formula:

$$(x_3 \vee \overline{x_1}) \wedge (x_2 \vee \overline{x_3}) \wedge (\overline{x_2} \vee \overline{x_1})$$

- (a) For the formula above, draw the associated graph.
- (b) Find a valid labeling of the vertices in this graph, or explain why no such labeling exists.