

Problem Set 7

Out: April 18, 2022

Due: April 26, 2022 at 11:59 pm EST

Instructions

Write your solution to the following problems carefully. Submit the PDF of your solution via Gradescope. Please start your solution to every question on a new page. Make sure to assign the correct page in your document corresponding to each problem. We recommend writing your solution in Latex. Handwritten solutions are accepted if they are clearly legible. Write your name and SID in your answer to Problem 0; do not write them anywhere else in your solution.

Collaboration policy: You may collaborate and discuss these problems with other students. However, you must first make an honest effort to solve these problems by yourself. You may discuss hints and try to solve problems together. But you may not explicitly provide the solution to a problem to other students; nor receive the solution from them. You must write your solution independently, and make sure you understand your solution. You must list all your collaborators (anyone with whom you discussed any part of these problems). You may consult the course textbook and other references related to the class. However, you are forbidden from searching for these problems on the Internet. You must list any resources you consult (beyond the course textbook). You must also follow the Yale academic integrity policy.

0 Your Information

On the first page of your submission, include the following information (but do not include these anywhere else in your solution). Also certify that you have followed the collaboration policy. Your solution to the remainder of the problems should start on a new page.

- (a) Your name.
- (b) Your SID.
- (c) A list your collaborators and any outside resources you consulted for this problem set. If none, write “None”.
- (d) Certify that you have followed the academic integrity and collaboration policy as written above.
- (e) How many hours did you spend in this problem set?

1 Strongly Independent Set

For an undirected graph $G = (V, E)$, a **strongly independent set** is a set $S \subseteq V$ of vertices such that for any two vertices $u, v \in S$, there is no path of length ≤ 2 between u and v .

(The length of a path is the number of edges in that path. Note that an *independent set* is a subset $S \subseteq V$ such that for any $u, v \in S$, there is no path of length 1 (i.e. an edge) between them.)

Consider the following problem **Strongly Independent Set (SIS)**:

- **Input:** An undirected graph $G = (V, E)$ and an integer $k > 0$
- **Output:** **Yes** if G has a strongly independent set of size k , else output **No**

Prove that SIS is NP-complete.

2 Dominating Set

For an undirected graph $G = (V, E)$, a **dominating set** is a set $D \subseteq V$ of vertices such that for every $v \in V$, v is either in D or adjacent to at least one member of D (i.e. there exists $u \in D$ such that $(u, v) \in E$).

Consider the following problem **Dominating Set (DS)**:

- **Input:** An undirected graph $G = (V, E)$ and an integer $k > 0$.
- **Output:** **Yes** if G has a dominating set of size at most k , else output **No**

Prove that this problem DS is NP-complete.